A flexible finite element tool for digital twin services

The 2025 INTO-CPS overture summit

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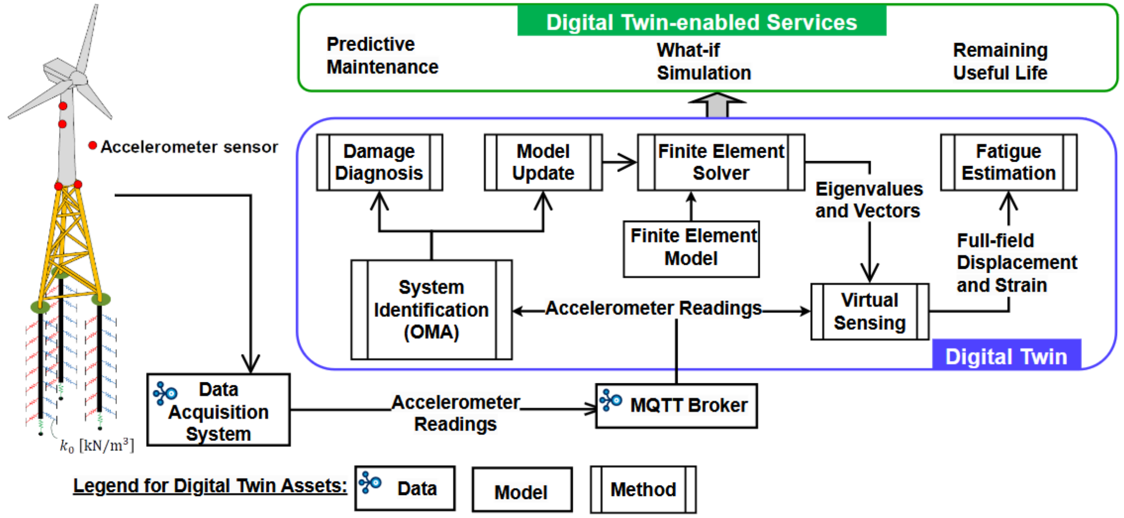
INTO-CPS

Introduction: Background and motivation

Goal of the research project is to develop a digital twin for structural system with flexible moving parts

<u>Digital Twin (DT) solution</u>
- A DT is a digital representation of a physical object, that through data exchange, reflects evolution of a physical twin over time

<u>What is required?</u>
- A flexible framework, which simulates the structural systems
- Finite element solver combined with multibody dynamical framework



DT framework(P. Talasila, 2025)

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Introduction: Scope

To address these challenges, this preliminary work presents a

- A from-scratch implementation that seamlessly integrates flexible body dynamics and FEM.
- A framework designed with a high degree of mobility in mind, prioritizing modularity, maintainability, compatibility and ease of distribution
- Addresses key challenges in computational performance, numerical stability, and software architecture, with applications spanning mechanical systems and input- state estimations requiring mid to high-fidelity dynamic analysis.

Outline

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YaFEM: Theoretical basis

System of equations

$$\begin{cases} \mathbf{M}\mathbf{a} + \mathbf{C}_p\mathbf{v} + \mathbf{r}(\mathbf{u}, \mathbf{v}, \mathbf{g}_q(t)) = \mathbf{B}_f\mathbf{g}_f(t) + \mathbf{B}_u\mathbf{g}_l(t) \\ \mathbf{B}_u^T\mathbf{u} = \mathbf{g}_u(t) \end{cases}$$

Where;

$\mathbf{M}, \mathbf{C}_p$ are the mass and proportional damping matrix

$\mathbf{u}, \mathbf{v}, \mathbf{a}$ are the displacement, velocity and acceleration vector

$\mathbf{g}_q, \mathbf{g}_f, \mathbf{g}_l$ are the temperature, force and displacement history vector

$\mathbf{r}$ is the restoring force vector

$\mathbf{B}_u, \mathbf{B}_f$ are the collocation matrices for controlled displacement and controlled forces

Proportional damping using coefficients

$$\mathbf{C}_p = \mathbf{M}\alpha + \mathbf{K}\beta$$

Mass, damping and stiffness matric computed using collocation matrices

$$\mathbf{M} = \sum_e^n \mathbf{Z}_{u,e}^\top \mathbf{M}_e \mathbf{Z}_{u,e} \quad , \quad \mathbf{C} = \sum_e^n \mathbf{Z}_{u,e}^\top \mathbf{C}_e \mathbf{Z}_{u,e} \quad , \quad \mathbf{K} = \sum_e^n \mathbf{Z}_{u,e}^\top \mathbf{K}_e \mathbf{Z}_{u,e}$$

Response matrix of displacement, velocity and temperature

$$\mathbf{u}_e = \mathbf{Z}_{u,e}\mathbf{u} \quad , \quad \mathbf{v}_e = \mathbf{Z}_{u,e}\mathbf{v} \quad , \quad \mathbf{q}_e = \mathbf{Z}_{q,e}\mathbf{g}_q(t)$$

Modal analysis

$$\left(\mathbf{K} - \omega_{m,i}^2\mathbf{M}\right)\Phi_{m,i} = 0$$

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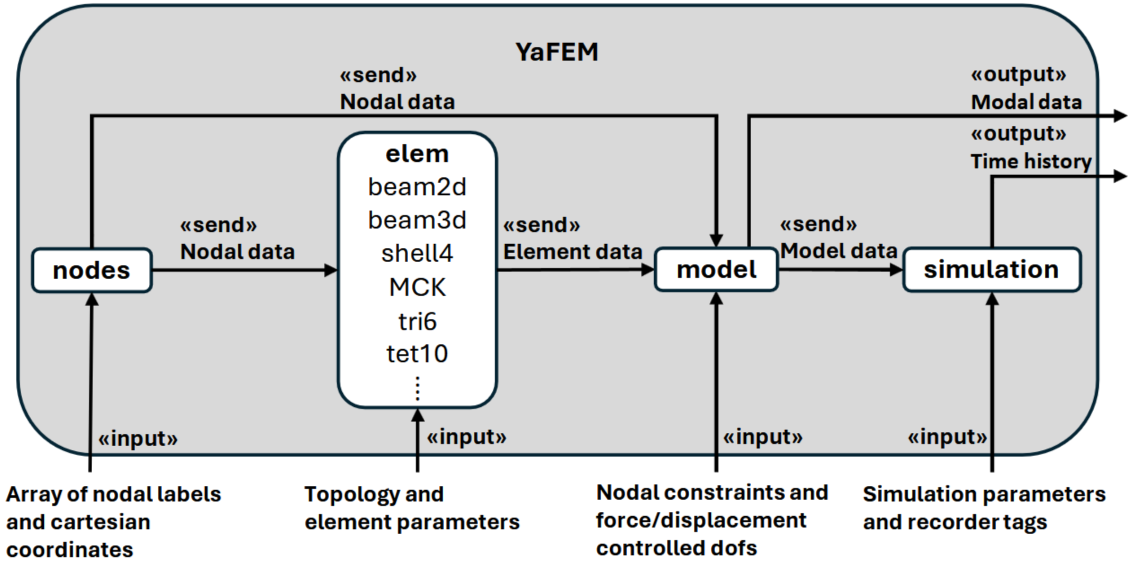
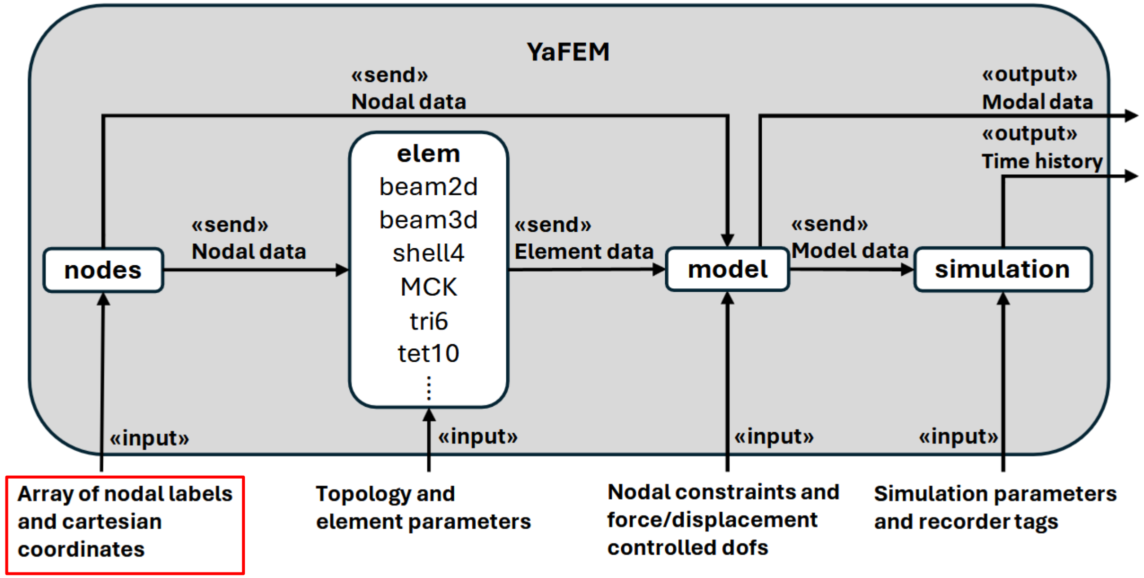
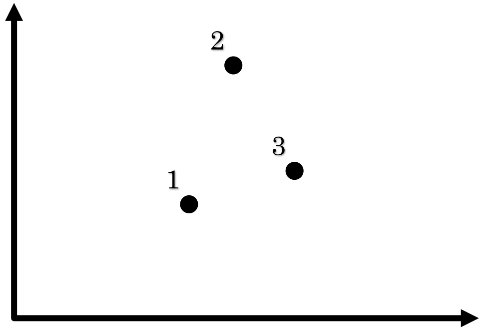
YaFEM: Architecture

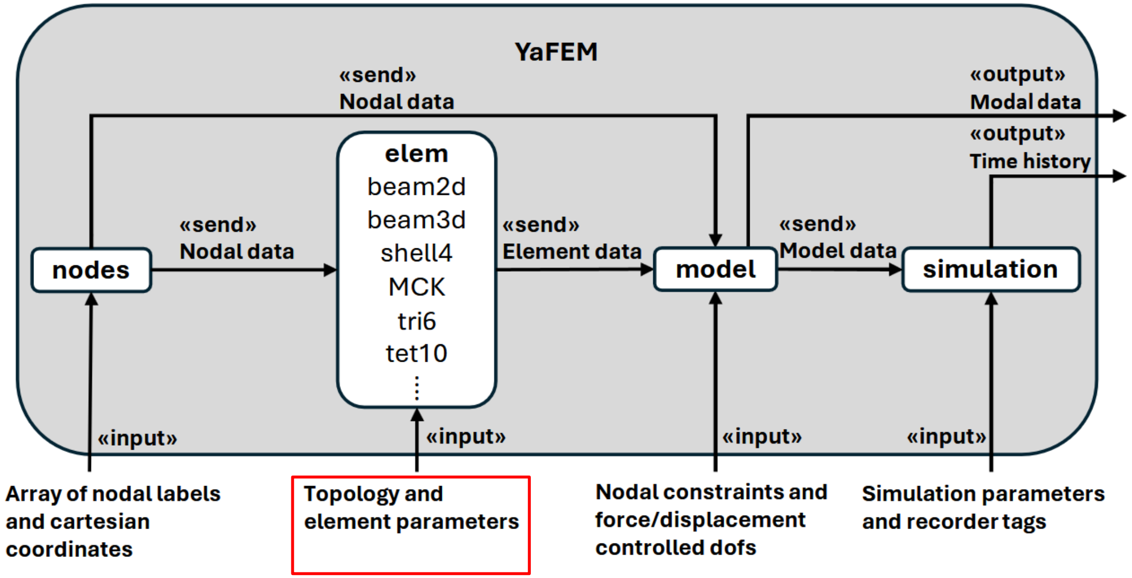


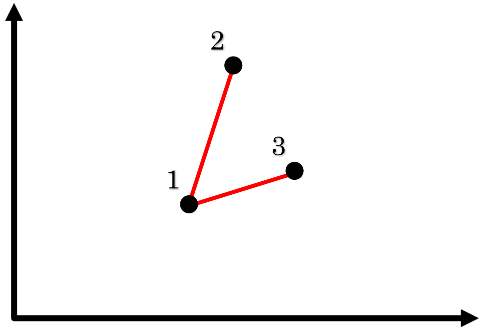
YaFEM: Architecture



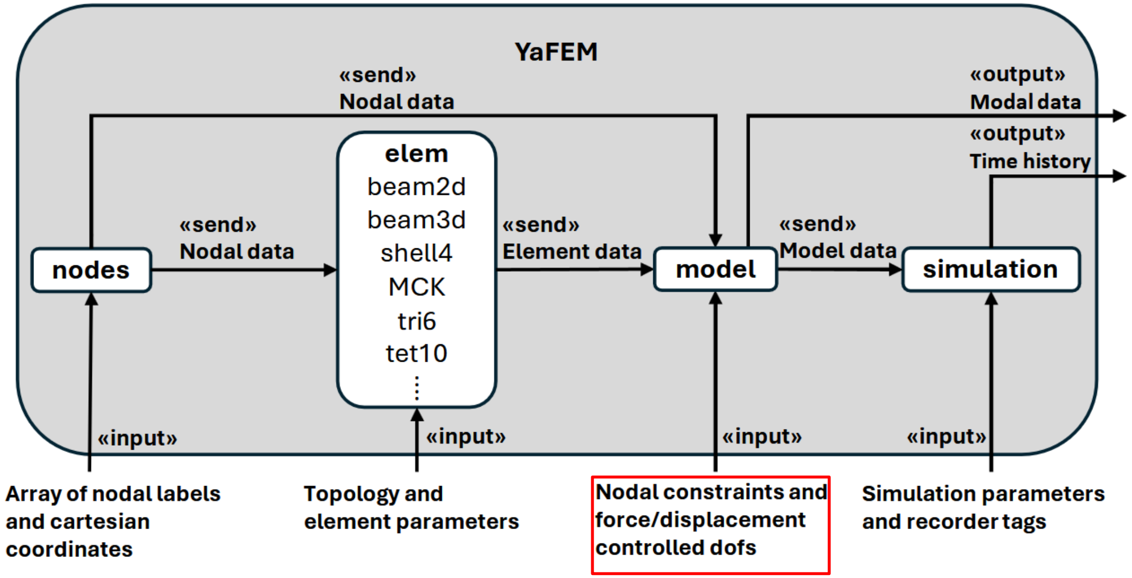


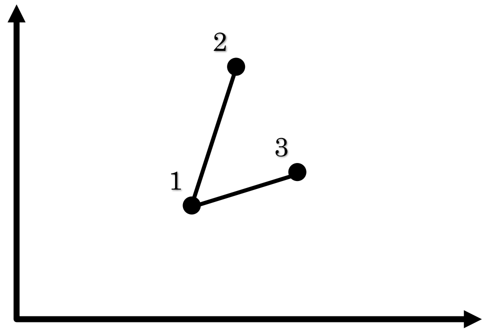
YaFEM: Architecture





YaFEM: Architecture





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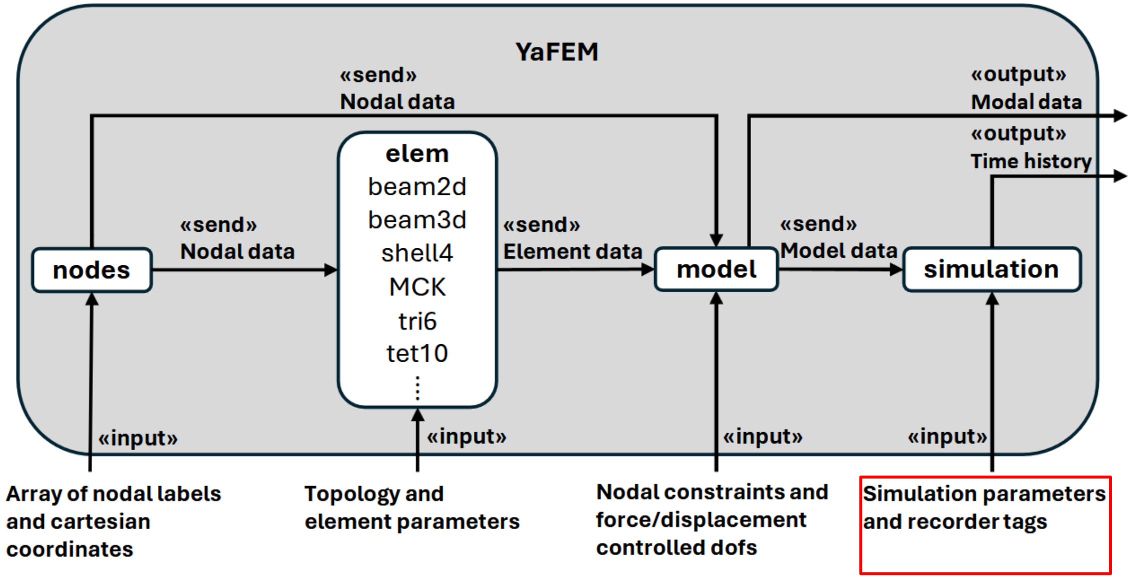
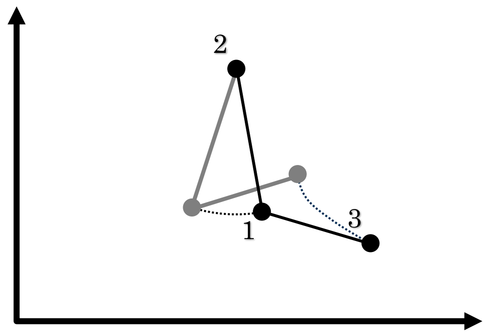
YaFEM: Architecture





YaFEM: Software comparisons

- Matlab
- Julia
- Python

Feature	Python	Julia	Matlab
Open source	Yes	Yes	No
Ease of maintenance	High (OOP supported)	Moderate	High (OOP supported)
Speed	Slow	Fast	Fast
Distribution	Easy	Easy	Limited
Community	Large	Growing	Moderate

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YaFEM: Tech stack

Symbolic expressions
- Sympy

Numerical expressions
- Numpy
- Scipy
- Jax

Graphical representations
- Matplotlib

Python standard library
- JSON
- Concurrent

YaFEM: Benchmark

- Cyclomatic complexity
- Halstead complexity
- Maintainability index

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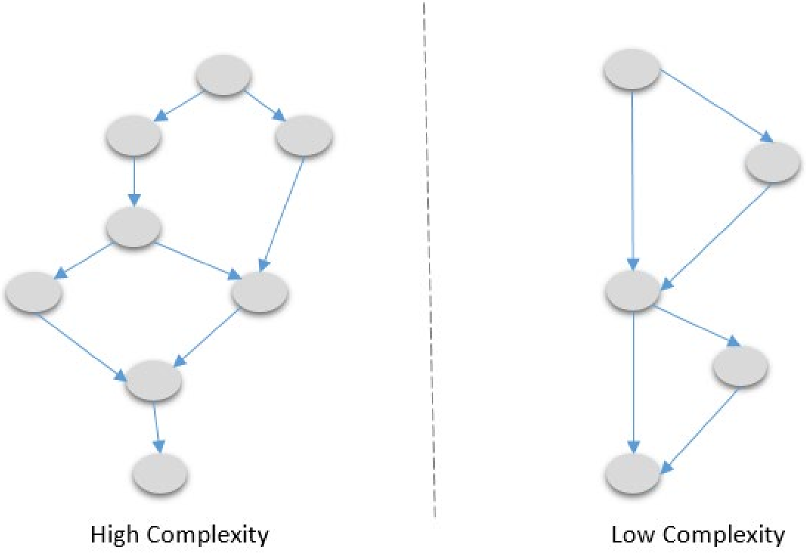
YaFEM: Benchmark

Cyclomatic complexity

$$CC = E - N + 2P$$

Where;

- E is the number of edges in the control flow graph
- N is the number of nodes in the control flow graph
- P is the number of connected components



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YaFEM: Benchmark

Halstead complexity

Metric	Meaning	Formula
Vocabulary	Number of unique operators and operands	$h = h_1 + h_2$
Program Length	Occurrences of operators and operands	$N = N_1 + N_2$
Program Length	Theoretical optimal program length	$\hat{N} = h_1 \log_2 h_1 + h_2 \log_2 h_2$
Volume	Total information content in the program	$V = N \cdot \log_2 h$
Difficulty	Complexity of understanding the code	$D = \frac{h_1}{2} \cdot \frac{N_2}{h_2}$
Effort	Effort required to write or understand	$E = D \cdot V$
Time	Estimated time (seconds) to understand	$T = \frac{E}{18}$
Estimated Bugs	Approximate number of potential bugs	$B = \frac{V}{3000}$

YaFEM: Benchmark

Maintainability index

$$MI = \max \begin{cases} 0 \\ (171 - 5.2 \cdot \ln(V) - 0.23 \cdot (CC) - 16.2 \cdot \ln(LOC)) \cdot 100/171 \end{cases}$$

Index Value	Meaning
0-9	Very high maintenance effort
10-19	Moderate maintenance effort
20-100	Low or acceptable maintenance effort

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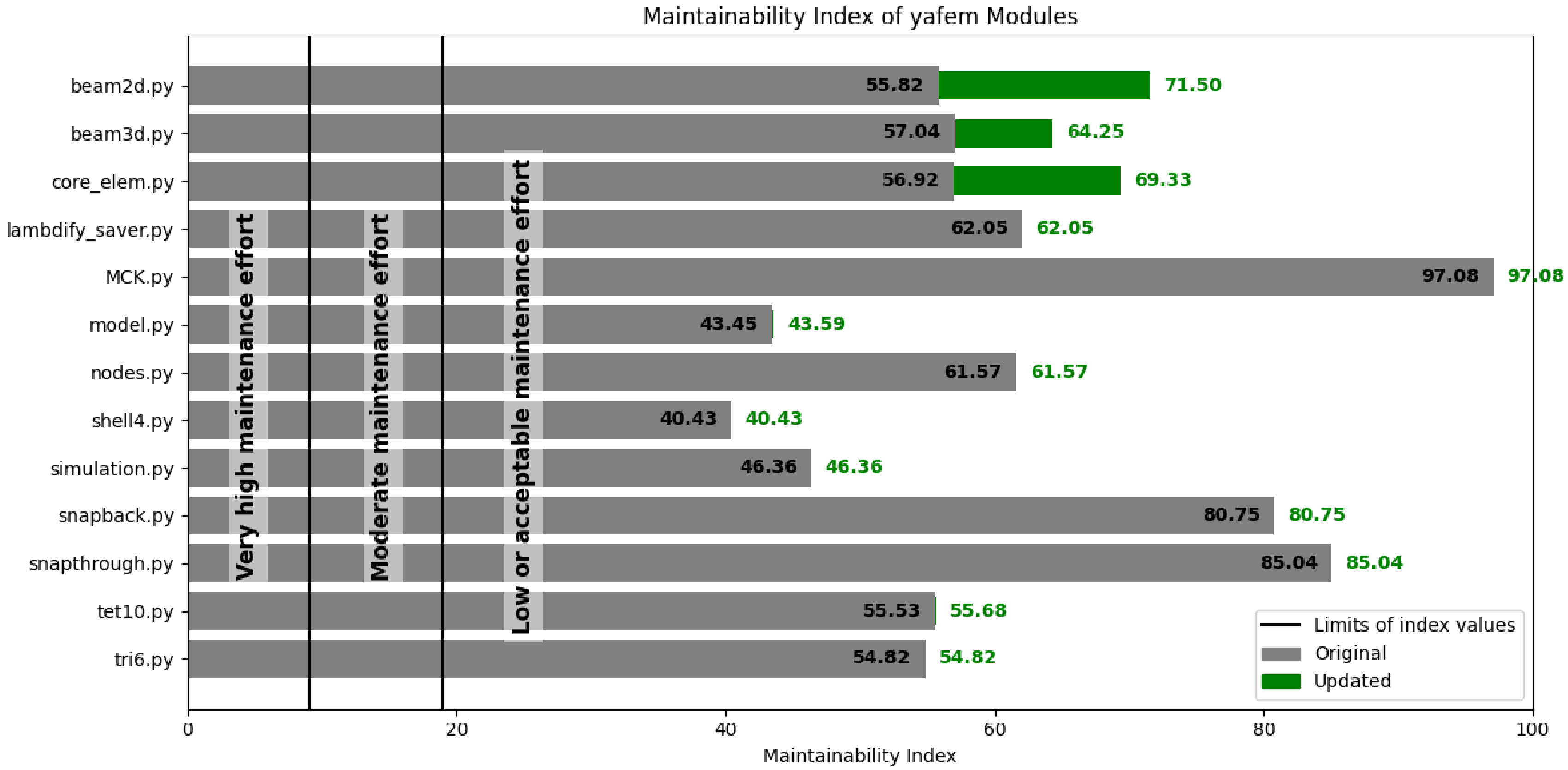
YaFEM: Benchmark

Maintainability index



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Use cases

- Offshore wind turbine model

  P. Talasila, D. Tcherniak, A.M.D. Jensen, S. Mahato, A. Sch¨orghofer-Queiroz, M.D. Ulriksen, G. Abbiati, P.G. Larsen, L. Damkilde: ”Structural Health Monitoring of Engineering Structures Using Digital Twins: A Digital Twin Platform Approach”

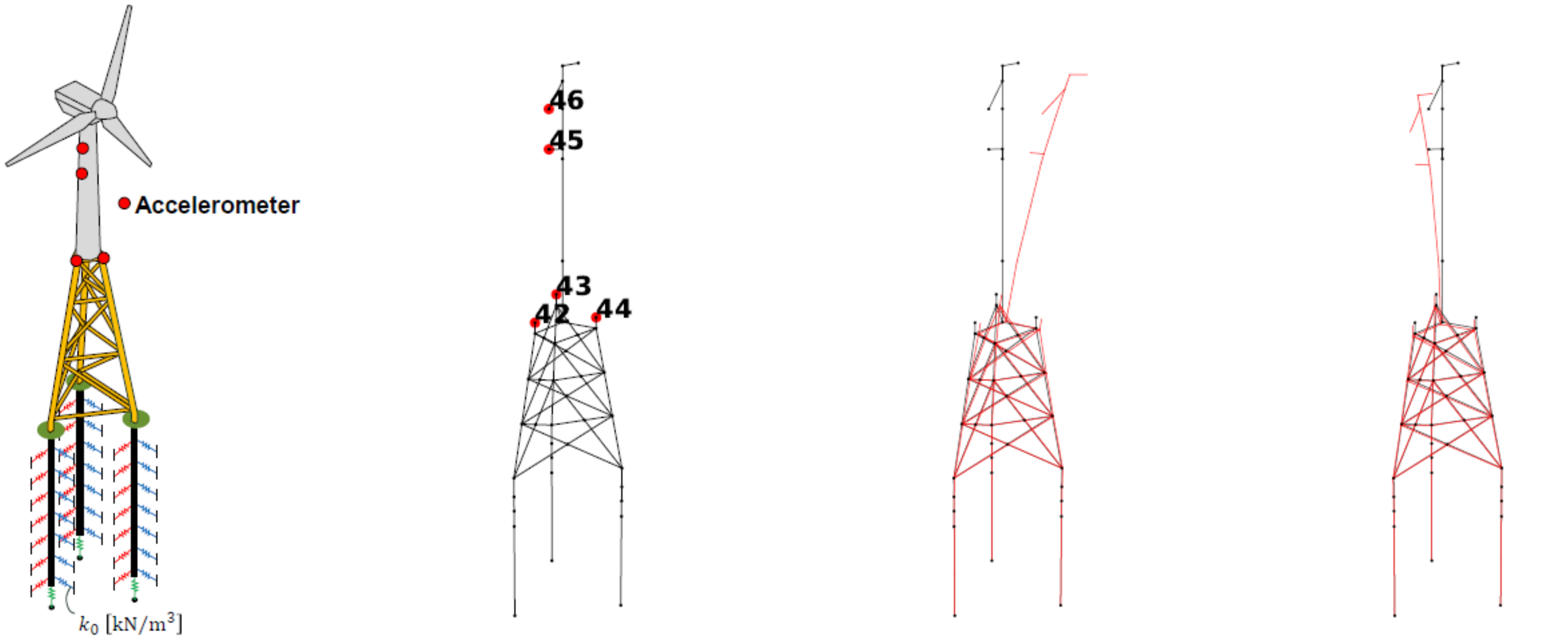
  Accepted and submitted paper for EVACES 2025 - International Conference On Experimental Vibration Analysis for Civil Engineering Structures

- Cooler model

  Part of CP-SENS project

Offshore wind turbine model

- Aim
- Modelling
  - Beam elements
  - MCK element (RNA element)



(a) System sketch. (b) Sensor locations. (c) Eigenmode 1 (d) Eigenmode 2

Eigenmode	1	2	3	4	5	6	7	8	9	10
Frequency (Hz)	0.20	0.29	1.33	1.35	1.36	2.34	2.34	3.66	4.06	4.26

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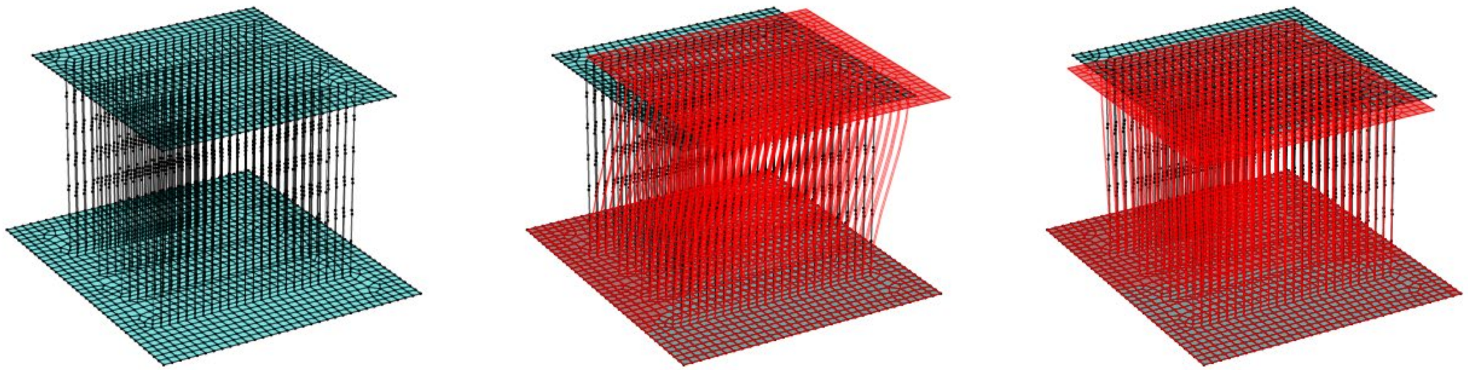
Cooler model

- Modelling
- Served as a great case study to optimize YaFEM
  - Speed
  - Scalability
  - Mixed elements
  - Compatibility



(a) Model plot (b) Eigenmode 1 (c) Eigenmode 2

Eigenmode	1	2	3	4	5	6	7	8	9	10
Frequency (Hz)	22.5	22.7	139.6	200.8	201.2	201.3	201.6	201.7	201.7	201.8

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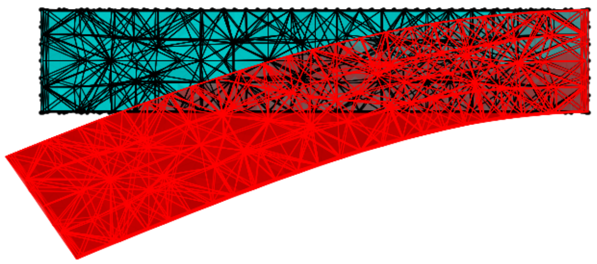
Future Work

YaFEM

- 6 noded triangle shell element (tri6)
- 10 noded tetrahedron volume element (tet10)
- Upload YaFEM to PYPI
- Add warm-start strategy
- Add increment matrix update strategy



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